

# ACCELERATION OF RELATIVISTIC ELECTRONS BY MHD TURBULENCE: IMPLICATIONS FOR NON-THERMAL EMISSION FROM BLACK HOLE ACCRETION DISKS

JACOB W. LYNN<sup>1,2</sup>, ELIOT QUATAERT<sup>2</sup>, BENJAMIN D. G. CHANDRAN<sup>3</sup>, & IAN J. PARRISH<sup>4</sup>

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## ABSTRACT

We use analytic estimates and numerical simulations of test particles interacting with magnetohydrodynamic (MHD) turbulence to show that subsonic MHD turbulence produces efficient second-order Fermi acceleration of relativistic particles. This acceleration is not well-described by standard quasi-linear theory but is a consequence of resonance broadening of wave-particle interactions in MHD turbulence. We provide momentum diffusion coefficients that can be used for astrophysical and heliospheric applications and discuss the implications of our results for accretion flows onto black holes. In particular, we show that particle acceleration by subsonic turbulence in radiatively inefficient accretion flows can produce a non-thermal tail in the electron distribution function that is likely important for modeling and interpreting the emission from low luminosity systems such as Sgr A\* and M87.

*Subject headings:* plasmas – heating – acceleration of particles – accretion, accretion disks

## 1. INTRODUCTION

In the limit of low-frequency magnetohydrodynamic (MHD) fluctuations, charged relativistic particles are accelerated by mirror forces resulting from magnetic compressions (Achterberg 1981),

$$\frac{dp_{\parallel}}{dt} = \frac{p_{\perp} v_{\perp}}{2B} \nabla_{\parallel} |B|, \quad (1)$$

where  $\parallel$  and  $\perp$  denote directions relative to the local magnetic field. In MHD, magnetic compressions are caused by slow modes and fast modes, with slow modes containing most of the compressive energy in subsonic turbulence. Because slow modes propagate approximately along the magnetic field in most regimes, a pure linear resonance with relativistic particles requires  $\omega \simeq k_{\parallel} v_p = k_{\parallel} v_{\parallel}$ , or equivalently  $v_p \simeq c$ , where  $v_p$  is the parallel phase velocity of slow modes. Thus linear theory predicts no acceleration of high-energy particles by MHD-scale slow modes, because the resonance condition cannot be satisfied. As a result, fast modes have traditionally been believed to be the dominant source of relativistic particle acceleration by MHD-scale turbulent fluctuations (Achterberg 1981; Miller et al. 1996). However, subsonic turbulence does not contain significant fast mode energy (see, e.g., Yao et al. 2011; Howes et al. 2012 for empirical constraints on the fast and slow mode energy in the solar wind). This appears to significantly limit the efficiency of relativistic particle acceleration by MHD turbulence in many astrophysical environments.

In strong MHD turbulence, the waves comprising MHD turbulence are not long-lived but instead have a decay

time comparable to their linear period. In this case, the linear resonance is not the appropriate condition for wave-particle interaction. Instead, the resonance is nonlinearly broadened (Bieber et al. 1994; Gruzinov & Quataert 1999; Shalchi et al. 2004; Shalchi & Schlickeiser 2004; Qin et al. 2006; Yan & Lazarian 2008; Lynn et al. 2012). Resonance broadening allows waves to interact with relativistic particles when  $\omega_{nl} \gtrsim k_{\parallel} c$ , where  $\omega_{nl}^{-1}$  is the non-linear correlation time of the turbulence. In this paper, we estimate the resulting particle acceleration analytically (§2) and numerically using simulations of relativistic test particles interacting with MHD turbulence (§3 & 4). Our results are potentially relevant to a wide range of astrophysical plasmas; in §5 we briefly assess the implications of our results for non-thermal emission from accretion disks around black holes.

## 2. RELATIVISTIC MOMENTUM DIFFUSION BY LOW-FREQUENCY MHD TURBULENCE

We first provide an order of magnitude estimate of the momentum diffusion coefficient for relativistic particles interacting with magnetic field compressions associated with slow modes (the case of fast modes is considered separately). The diffusion coefficient for a particle with reduced momentum  $\bar{p} \equiv p/mc$  may be estimated as  $D_{\bar{p}, \mathbf{k}} \sim f^2 \delta t / c^2$  where  $f \sim \bar{p} c^2 k_{\parallel} \delta B_{\parallel}(\mathbf{k}) / B_0$  is the force felt by a particle interacting with a given spatial scale labeled by  $\mathbf{k}$ ,  $\delta t \sim \omega_{nl}^{-1}$  refers to the timescale over which wave-particle interactions are correlated, and  $\delta B_{\parallel}(\mathbf{k})$  is the rms fluctuation in magnetic compressions on scale  $\mathbf{k}$ . For a given  $k_{\perp}$ , the total acceleration will be determined by the average of  $k_{\parallel}^2 \delta B_{\parallel}(\mathbf{k})^2$  over  $k_{\parallel}$ , limited to those  $k_{\parallel}$  that satisfy the broadened resonance condition  $k_{\parallel} \lesssim \omega_{nl}/c$ . Provided that  $\delta B_{\parallel}(\mathbf{k})^2$  does not scale too steeply with  $k_{\parallel}$ , parallel wavenumbers near  $k_{\parallel} \sim \omega_{nl}/c$  will dominate, resulting in a diffusion coefficient at fixed  $k_{\perp}$  of order

$$D_{\bar{p}, k_{\perp}} \sim \bar{p}^2 \frac{v_A}{c} \frac{\omega_{nl} \delta B_{\parallel}(\mathbf{k}_{\perp})^2}{B_0^2}. \quad (2)$$

<sup>1</sup> Physics Department, University of California, Berkeley, CA 94720; jacob.lynn@berkeley.edu

<sup>2</sup> Astronomy Department and Theoretical Astrophysics Center, University of California, Berkeley, CA 94720

<sup>3</sup> Space Science Center and Department of Physics, University of New Hampshire, Durham, NH 03824

<sup>4</sup> Canadian Institute for Theoretical Astrophysics, 60 St. George Street, University of Toronto, Toronto, ON M5S 3H8, Canada

where we have used the fact that most of the turbulent energy in anisotropic MHD turbulence has  $k_{\parallel} \lesssim \omega_{\text{nl}}/v_A$  and that only a fraction  $v_A/c$  of the energy in magnetic compressions satisfies the conditions required for efficient particle acceleration, namely  $k_{\parallel} \lesssim \omega_{\text{nl}}/c$  (**this follows formally from the magnetic field power spectrum in eq. 5 below, which implies a parallel energy spectrum of  $dE/d\ln k_{\parallel} \propto k_{\parallel}$** ). Equation 2 shows that the scaling of  $\omega_{\text{nl}}\delta B_{\parallel}(\mathbf{k}_{\perp})^2$  with  $k_{\perp}$  determines which  $k_{\perp}$  dominates. For the strong MHD power spectrum of Goldreich & Sridhar (1995),  $\omega_{\text{nl}} \propto k_{\perp}^{2/3}$  and  $\delta B(\mathbf{k}_{\perp}) \propto k_{\perp}^{-1/3}$ , so that all scales contribute equally, provided that they can satisfy  $k_{\parallel} \lesssim \omega_{\text{nl}}/c$  (which favors long-wavelength fluctuations).

More formally, the resonance-broadened diffusion coefficient for the reduced momentum is given by (Dupree 1966; Weinstock 1969)

$$D_{\bar{p}_{\parallel}} = \frac{\bar{p}_{\perp}^2 v_{\perp}^2}{4B_0^2} \int d^3k k_{\parallel}^2 I_B(\mathbf{k}) R(\mathbf{k}), \quad (3)$$

where  $\bar{p}_{\perp}$  is the perpendicular component of the dimensionless momentum,  $I_B(\mathbf{k})$  is the 3D power spectrum of magnetic field fluctuations, and  $R(\mathbf{k})$  is a resonance function that describes the time-averaged interaction of a test particle with waves at a given  $\mathbf{k}$ . Quantitatively,  $R(\mathbf{k}) = \Re \int_0^{\infty} dt \exp[i(\omega(\mathbf{k}) - v_{\parallel}k_{\parallel})t] f(t)$ , where  $f(t)$  is the time correlation function for wave-particle interactions.  $R(\mathbf{k})$  is necessarily phenomenological, as it in principle depends on the momentum diffusion coefficient itself. Standard models in the literature assume that waves decay as an exponential or Gaussian in time due to non-linear interactions; e.g.  $f(t) = e^{-\omega_{\text{nl}}^2 t^2}$  (a Gaussian decay model), with a nonlinear decay frequency  $\omega_{\text{nl}}$ . We focus on a Gaussian decay model favored by our previous test particle simulations (Lynn et al. 2012). These assumptions lead to

$$R(\mathbf{k}) = \frac{\sqrt{\pi}}{2\omega_{\text{nl}}} \exp\left[-\frac{k_{\parallel}^2(v_{\parallel} - v_p)^2}{4\omega_{\text{nl}}^2}\right]. \quad (4)$$

where we have assumed for simplicity that the waves have a dispersion relation  $\omega = k_{\parallel}v_p$ , a reasonable approximation for anisotropic slow modes with  $k_{\perp} \gg k_{\parallel}$ . Physically, equation 4 implies that for the particles to couple to the turbulent fluctuations, the frequency that the particles feel as they pass through a wave,  $k_{\parallel}(v_{\parallel} - v_p)$ , must be of order (or less than) the nonlinear frequency.

To perform the calculation in Equation 3, we assume that the magnetic power spectrum of slow modes is given by strong anisotropic turbulence,

$$I(\mathbf{k}) \equiv \frac{\delta B_S^2 L^3}{12\pi} (k_{\perp} L)^{-10/3} g\left(\frac{k_{\parallel} L^{1/3}}{k_{\perp}^{2/3}}\right), \quad (5)$$

where  $L$  is the outer scale of the cascade,  $\delta B_S^2$  is the total energy in slow mode magnetic fluctuations, and  $g(x) \simeq 1$  for  $x \lesssim 1$  and falls off sharply to zero for  $x \gtrsim 1$ . The cutoff  $g(x)$  in equation 5 represents the lack of power outside the anisotropic Goldreich-Sridhar cone (Goldreich & Sridhar 1995) in weakly compressible MHD turbulence,<sup>5</sup>

<sup>5</sup> Cho et al. (2002) show using numerical simulations

and the power spectrum normalization is chosen so that  $\int d^3k I(\mathbf{k}) \equiv \delta B_S^2/2$ . The nonlinear frequency in such turbulence is given by  $\omega_{\text{nl}} \simeq (v_A/L)(k_{\perp} L)^{2/3}$ , which is of order the eddy turnover time on a given scale.

Finally, to simplify the resulting estimate, we assume that the particles are relativistic ( $\bar{p}_{\perp} \sim \bar{p}_{\parallel} \gg 1$ ), and that wave speeds are nonrelativistic. To emphasize the net particle acceleration efficiency, we also present our results in terms of the total momentum diffusion coefficient, rather than the parallel momentum diffusion coefficient. This implicitly assumes that modest pitch angle scattering isotropizes the distribution function (see §4). Under these approximations, the diffusion coefficient becomes

$$D_{\bar{p}} = \frac{\sqrt{\pi}\bar{p}_{\perp}^2 v_{\perp}^2}{48v_A} \frac{\delta B_S^2}{B_0^2} \left(\frac{v_{\parallel}}{v}\right)^2 \times \int dk_{\parallel} dk_{\perp} \frac{k_{\parallel}^2}{k_{\perp}^3} g\left(\frac{k_{\parallel} L^{1/3}}{k_{\perp}^{2/3}}\right) \exp\left(-\frac{k_{\parallel}^2 L^2 v_{\parallel}^2}{4(k_{\perp} L)^{4/3} v_A^2}\right). \quad (6)$$

Both the exponential and the  $g(x)$  term in equation 6 have the effect of cutting off the interaction at high  $k_{\parallel}$  for a given  $k_{\perp}$ . When  $v_A \ll v_{\parallel}$ , the exponential cutoff will always be more constraining and requires  $k_{\parallel} \lesssim L^{-1}v_A/c(k_{\perp} L)^{2/3}$ . Given this, we find

$$D_{\bar{p}} = \bar{p}^2 \frac{\pi}{24} \frac{v_A^2}{cL} \frac{\delta B_S^2}{B_0^2} \frac{(1 - \alpha^2)^2}{\alpha^3} \ln(k_{\text{max}} L), \quad (7)$$

where we have rewritten the angular dependence in terms of  $\alpha = v_{\parallel}/v$ , the particle pitch-angle cosine. The result in equation 7 is similar to that derived earlier by Chandran (2000). The restriction to parallel velocities much larger than  $v_A$  corresponds to  $\alpha \gg v_A/v \simeq v_A/c$ . Equation 7 corresponds to a particle acceleration time  $t_a \sim (\delta B_S/B_0)^{-2}(L/v_A)(c/v_A)$ . Note that this is independent of particle energy and is roughly the eddy turnover time divided by the fraction of the magnetic energy at low  $k_{\parallel}$  that satisfies  $k_{\parallel} \lesssim \omega_{\text{nl}}/c$ .

Equation 7 can be compared to the analogous result for acceleration of relativistic particles by fast modes. The latter is predominantly via a linear resonance with highly oblique waves. Versions of this calculation have been performed in many other contexts (Miller et al. 1996; Yan & Lazarian 2002), so we restrict ourselves to briefly summarizing the salient features here.

Relativistic test particles travelling along magnetic field lines can experience a linear resonance with a highly oblique fast mode. The linear resonance function is a delta-function,

$$R(\mathbf{k}) = \pi\delta(kv_p \pm k_{\parallel}v_{\parallel}), \quad (8)$$

where  $v_p$  is the isotropic phase velocity of fast modes (approximately  $v_A$  for  $\beta \ll 1$  and  $c_s$  for  $\beta \gg 1$ ). The power spectrum of fast modes in Alfvénic turbulence is believed to be isotropic. The spectral index is uncertain but the simulations of Kowal & Lazarian (2010) suggest a 1D power spectrum  $P(k) \sim k^{-2}$  (though possibly shallower; see Cho & Lazarian 2003; Chandran 2005).

that  $g(x) \propto \exp[-x]$ . Our results are insensitive to the precise functional form of  $g(x)$ .

Performing the integral in Equation 3 with the fast mode linear resonance and isotropic power spectrum  $\sim \delta B_F^2 L^3 (kL)^{-\alpha}$  leads to

$$D_{\bar{p}} \sim \bar{p}^2 \frac{v_p^2}{cL} \frac{\delta B_F^2}{B_0^2} \int_{k_{\min}}^{k_{\max}} dk k^{1-\alpha}, \quad (9)$$

where  $\delta B_F^2$  is the energy in magnetic compressions associated with fast modes. For  $\alpha \sim 2$ , the diffusion coefficient due to fast modes (eq. 9) is of the same form as that due to slow modes (eq. 7). For  $\beta \gg 1$ , a comparison of these two expressions shows that ratio of the fast mode to slow mode diffusion coefficient is  $\sim (c_s/v_A)^2 (\delta B_F/\delta B_S)^2$ . At high  $\beta$ , fast modes lose their magnetic compressibility (becoming simply sound waves), so that the magnetic energy  $\delta B_F^2$  in fast modes decreases at fixed velocity amplitude, with  $\delta B_F^2 \sim \rho \delta v_F^2/\beta$ . By contrast,  $\delta B_S^2 \sim \rho \delta v_S^2$ . Thus the ratio of the fast to slow mode diffusion coefficients is in fact set by the relative turbulent energy in each mode. For subsonic turbulence, slow modes will in general dominate the particle acceleration because there is significantly more energy in slow modes. This depends, however, on the power spectrum of the fast modes. If  $\alpha \sim 3/2$  rather than  $\alpha \sim 2$  (as in Chandran 2005) then the fast mode acceleration efficiency can be greater than that of slow modes even given the overall lower energy density in fast modes.

### 3. NUMERICAL METHODS

Our simulations consist of charged test particles evolving in the macroscopic electric and magnetic fields of isothermal, subsonic MHD turbulence. Apart from modifying the particle pusher for relativistic test particles, which we describe below, our computational approach is identical to that of Lynn et al. (2012). Dimensional quantities throughout the paper are expressed in units of the sound speed  $c_s$  and the box scale  $L$ , when not explicitly stated.

#### 3.1. Turbulence simulations

We simulate ideal MHD turbulence with the Athena code (Stone et al. 2008). We drive an incompressible turbulent velocity field using an Ornstein-Uhlenbeck process, and allow compressible fluctuations to develop naturally. The OU process has a characteristic autocorrelation time  $t_{OU}$ . Fiducial properties for the MHD simulations used in this work are summarized in Table 1. We show results from higher resolution calculations in Figure 3 (discussed below) and find that the numerically determined diffusion coefficients are relatively independent of resolution. In our calculations, the simulation box is extended along the mean magnetic field, because otherwise the particles (which undergo periodic boundary conditions) would interact with the same eddies multiple times before the eddies decorrelate.

##### 3.1.1. Measurement of turbulence properties

An important property of our turbulence simulations for comparing test particle results to analytical estimates is the rms deviation in parallel magnetic field,  $\delta B_{\parallel}$ , since this sets the magnitude of the magnetic mirror forces. We define this quantity as the spatial rms average of  $|\mathbf{B}| - |\mathbf{B}_0|$ , where  $\mathbf{B}_0$  is the initial mean magnetic field.

TABLE 1  
SUMMARY OF FIDUCIAL SIMULATION PROPERTIES

Parameter	Value
Resolution	$512 \times 128^2$
Volume ( $L^3$ )	$8 \times 2^2$
$\epsilon$ ( $c_s^3/L$ ) <sup>a</sup>	0.1
$\beta$ <sup>b</sup>	1
$t_{OU}$ ( $L/c_s$ ) <sup>c</sup>	1.5
$l_D$ ( $L$ ) <sup>d</sup>	0.39
$\delta B_{\parallel}$ ( $B_0$ ) <sup>e</sup>	0.12
$N_{\text{particles}}$	$2^{11} \times 10^3 \simeq 2 \times 10^6$
$\Omega_0$ ( $c_s/L$ ) <sup>f</sup>	$2 \times 10^5$

<sup>a</sup>The turbulent energy input rate, corresponding to a sonic Mach number of  $\simeq 0.35$ . Calculations with  $\epsilon = 0.01$  yield similar results.

<sup>b</sup>Ratio of thermal to magnetic pressure. Our calculations covered a range of  $\beta \sim 0.1 - 10$ .

<sup>c</sup> $t_{OU}$  refers to the correlation time in the Ornstein-Uhlenbeck turbulence forcing.

<sup>d</sup>Outer (driving) scale of the turbulence.

<sup>e</sup>RMS fluctuation in the parallel magnetic field.

<sup>f</sup>Test particle gyrofrequency.

This is equivalent to taking the local direction of the magnetic field as the “parallel” direction. The magnitude of  $\delta B_{\parallel}$  depends on the magnetic compressibility of the fast and slow modes at a given  $\beta$ , in addition to their overall representation in the turbulence. For our fiducial simulation summarized in Table 1,  $\delta B_{\parallel}/B_0 \simeq 0.12$ , while simulations that have the same driving rate (and thus similar  $v_{\text{rms}}$ ) but  $\beta = 0.3$  ( $\beta = 3$ ) have  $\delta B_{\parallel}/B_0 \simeq 0.05$  (0.23). Note that for higher  $\beta$ , the energy in magnetic compressions is larger.

We also decompose the turbulent velocity field into the linear MHD Alfvén, slow, and fast modes, following the approximate Fourier space method of Cho & Lazarian (2003). For our fiducial simulation, 50%, 45%, and 5% of the turbulent energy is in the Alfvén, slow, and fast modes respectively. For both higher and lower  $\beta$ , the proportion of energy in fast modes decreases substantially (to less than 1%), while the slow mode energy remains at the same order of magnitude. Thus it is broadly appropriate to assume that all of the magnetic field fluctuation energy is in the slow modes, and that the particle acceleration is dominated by interactions with slow modes.

#### 3.2. Test particle integration

For a given fluid simulation, we simulate a statistical ensemble of charged test particles which are initialized randomly throughout the box of fully saturated turbulence. These test particles are evolved according to the Lorentz force

$$\frac{d\bar{\mathbf{p}}}{dt} = \frac{q}{mc} \mathbf{E} + \frac{q}{mc} \boldsymbol{\beta} \times \mathbf{B}, \quad (10)$$

where the dimensionless momentum  $\bar{\mathbf{p}} \equiv \mathbf{p}/mc$ , and  $\boldsymbol{\beta} = \mathbf{u}/c$  is the particle’s physical velocity. The  $\mathbf{E}$  and  $\mathbf{B}$  fields are those on the MHD grid, interpolated to the particle’s location using the triangular-shaped cloud (Hockney & Eastwood 1981) method in space and time. For each simulation, we also choose a numerical value for the speed of light  $c$ , which affects the motion of the test particles. The choice of  $c$  does not, however, affect the turbulence. Our choice of non-relativistic turbulence and

relativistic test particles is appropriate for studying high energy supra-thermal particle acceleration.

Particles are integrated using the Vay (2008) particle pusher, which is symplectic and symmetric in time, and conserves energy and the magnetic moment adiabatic invariant to machine precision in tests with constant fields.<sup>6</sup> We initialize the test particles with sufficiently high gyrofrequencies that diffusion and heating is independent of gyrofrequency; i.e.  $\Omega \gg \omega$  where  $\omega$  is the frequency of any turbulent motions and  $\Omega$  is the relativistic gyrofrequency. To calculate the momentum diffusion coefficients, we further initialize particles with a specific value of  $\bar{p}$ , and generally take  $\bar{p}_\perp = \bar{p}_\parallel$ . The momentum is defined with respect to the bulk rest-frame of the simulation, though for the relativistic particles we focus on, this choice is unimportant because the particle velocities are much greater than the fluid velocities.

The velocity diffusion coefficients are calculated according to

$$D_{\bar{p}} \equiv \frac{\langle \delta \bar{p}^2 \rangle}{2 \delta t}, \quad (11)$$

where the average is over many particles with the same initial momentum. One subtlety is that because we initialize the particle momentum with respect to the bulk rest frame of the simulation, they are not initially moving with the local drift velocity. As they change their motion to follow the drift velocity, the particle momentum undergoes an initial transient “jump” which saturates at the rms velocity of the turbulence (times the Lorentz factor of the test particle, for ultra-relativistic particles). We sidestep this subtlety by fitting a linear function in  $t$  to  $\langle \delta \bar{p}^2 \rangle$  at later times using least-squares.

#### 4. NUMERICAL RESULTS

In the analytic calculations summarized in §2, the particles are assumed to diffuse primarily in  $p_\parallel$ , as expected for particles interacting with long wavelength, low frequency turbulent fluctuations. In our numerical calculations, we find that particles undergo diffusion in both  $p_\parallel$  and  $p_\perp$  (or, equivalently, in total momentum  $p$  and magnetic moment  $\mu$ ). This is true for both the relativistic calculations presented here and our earlier non-relativistic test particle calculations (Lehe et al. 2009; Lynn et al. 2012). For the non-relativistic test particle calculations, the diffusion time for  $\mu$  was somewhat longer than that for the total momentum  $p$ , while for the relativistic test particle results presented in this paper, the two timescales are comparable. This diffusion in  $\mu$  corresponds to an effective pitch angle scattering rate and may be due to violation of magnetic moment conservation by finite amplitude low frequency turbulent fluctuations (Chandran et al. 2010). The theory for the latter has not been fully worked out for the  $\beta \sim 1$  conditions we focus on here. In what follows, we defer a detailed analysis of the diffusion in  $\mu$  to future work and focus on the diffusion in total momentum  $p$ .

Figure 1 demonstrates that for relativistic test particles, the momentum diffusion coefficient is robustly of

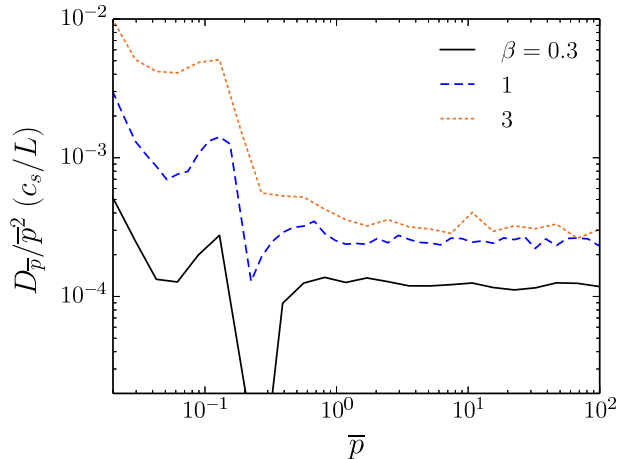


FIG. 1.— Test particle momentum diffusion coefficients as a function of  $\bar{p} \equiv p/mc$  normalized by  $\bar{p}^2$  for several values of  $\beta$  (for fixed driving rate  $\dot{\epsilon} = 0.01 c_s^3/L$  and fixed  $c = 10 c_s$ ). For each  $\bar{p}$ , the diffusion coefficient is measured for an ensemble of particles with  $\bar{p}_\perp = \bar{p}_\parallel = \bar{p}/\sqrt{2}$  that are initially at random positions in the turbulent box. The numerical results demonstrate that  $D_{\bar{p}} \propto \bar{p}^2$  for ultrarelativistic particles ( $\bar{p} \gg 1$ ), consistent with the analytic expectations from equation 7.

the form  $D_{\bar{p}} \propto \bar{p}^2$ . This is in contrast to the case of non-relativistic particles where the diffusion coefficient for particles interacting with subsonic turbulence is roughly  $D \propto p$  for supra-thermal particles (Lynn et al. 2012).

The analytic results summarized in equation 7 also predict that for relativistic particles, the magnitude of the diffusion coefficient depends on the magnetic compressibility of the turbulence and  $v_A/c$ . We now test these expectations using our test particle simulations.

Figure 2 shows how the measured diffusion coefficient (normalized by  $\bar{p}^2$ ) varies with the rms magnetic field compression  $\delta B_\parallel/B_0$ . The latter is directly measured in the simulations as described in §3.1.1. Figure 2 shows that the diffusion coefficient scales with the total turbulent energy in the magnetic field compressions, consistent with equation 7.

Figure 3 shows the dependence of the measured diffusion coefficient on the ratio  $c/v_A$ , for three different values of  $\beta$ . We reiterate that in each simulation, we choose a value for the speed of light  $c$  only for the purposes of evolving the test particles (the choice of  $c$  has no impact on the properties of the turbulence). The diffusion coefficients in Figure 3 are normalized by the analytic prediction in equation 7.<sup>7</sup> For each  $\beta$ ,  $v_A$  and  $c_s$  are fixed, so the x-axis in Figure 3 corresponds to different choices of  $c$ . For  $c \gg v_A$ , the results are reasonably consistent with the analytical predictions. In addition to the results for the fiducial simulation, Figure 3 also shows two other  $\beta = 1$  simulations: (1) one with the same resolution but a larger driving scale by a factor of two, so that the inertial range is somewhat more extended (HR) (2) a second higher resolution  $1024 \times 256^2$  simulation. The results for both of these other simulations are very similar to the fiducial calculation, confirming that the large-scale

<sup>6</sup> The Boris (1970) pusher is not as accurate when fluid velocities are non-negligible fractions of the chosen value of  $c$ . In tests, these errors did not significantly affect our results, but we nevertheless prefer the Vay pusher for the relativistic case.

<sup>7</sup> Specifically, we use  $D_{\bar{p}} \simeq 0.4 \bar{p}^2 (\delta B_\parallel/B_0)^2 v_A^2/cL$  for the analytic prediction from equation 7, with  $\delta B_\parallel/B_0$  calculated for each simulation, where we have used  $\ln(k_{\max}L) = \ln 64 \simeq 4.15$  for the fiducial simulation. For the higher resolution simulations, the logarithmic factor is adjusted as appropriate.

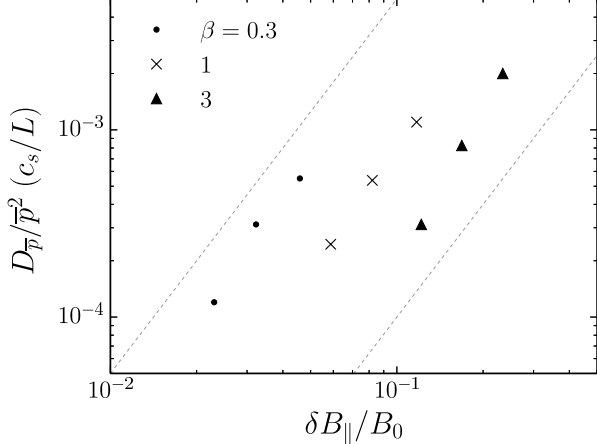


FIG. 2.— Test particle momentum diffusion coefficient for relativistic particles as a function of the strength of the magnetic compressions  $\propto \delta B_{\parallel}$ , for several values of  $\beta$ . The simulations have  $c = 10 c_s$  and different turbulent driving rates  $\epsilon$ . The numerically determined diffusion coefficients are  $\propto (\delta B_{\parallel}/B_0)^2$  (light dashed lines, with arbitrary normalization), consistent with the analytical predictions in §2.

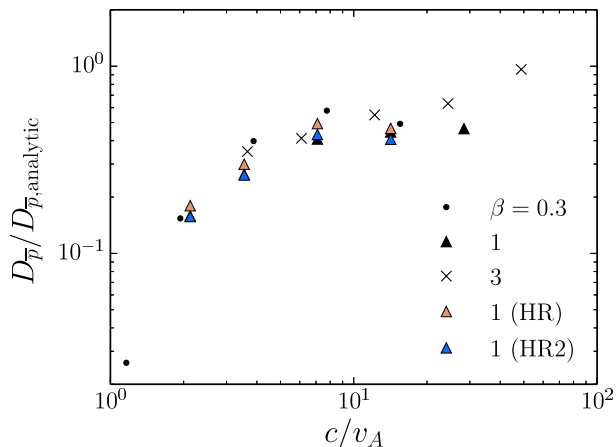


FIG. 3.— Test particle momentum diffusion coefficients for relativistic particles normalized by the analytic prediction of equation 7 for simulations with different  $\beta$  and  $v_A/c$ . For  $c \gg v_A$ , the results are well-described by the analytical predictions. The test particle calculations are for an ensemble of particles with  $\bar{p}_{\perp} = \bar{p}_{\parallel} = \bar{p}/\sqrt{2}$ . The orange triangles (HR) are from a run with the fiducial number of grid cells but a larger turbulent driving scale, so that the turbulence has an inertial range that is 2 times larger. The blue triangles (HR2) are for a higher resolution  $1024 \times 256^2$  simulation. Both resolution tests yield very similar results indicating that large scale turbulent fluctuations dominate the particle acceleration.

fluctuations that are well-resolved in a typical MHD simulation produce the majority of the particle acceleration.

#### 4.1. Long-time evolution of distribution function

The diffusion coefficients shown in Figure 3 correspond to particle acceleration times that are many eddy turnover times. As a result, it is computationally intensive to directly simulate the long timescale evolution of the distribution function. To study the latter, we instead separately solve the time dependent diffusion equation for the distribution function  $f(p, t)$  using the momentum diffusion coefficients determined in our test particle calculations. In particular, we begin with

a Maxwellian distribution function having  $k_B T \sim mc^2$ , i.e.,  $\langle \bar{p} \rangle \sim 1$  and evolve it subject to a diffusion coefficient given by  $D_{\bar{p}} \equiv \bar{p}^2/t_a$  where  $t_a$  defines the acceleration time. Figure 4 (*right panel*) shows the resulting distribution function at later times. For comparison, we also show a Maxwell-Jüttner distribution function (black dashed lines) that has the same total energy as the final distribution function in our diffusion calculations. Figure 4 shows that the distribution function quickly develops a significant non-thermal tail, on a timescale of  $\sim 0.25 t_a$ . As a consistency check, the *left panel* in Figure 4 shows that over the timescale we can directly simulate the MHD turbulence with test particles, the evolution of the distribution function is indistinguishable from the solution of the momentum-space diffusion equation.

## 5. CONCLUSIONS & IMPLICATIONS

Our results demonstrate that subsonic MHD turbulence efficiently accelerates relativistic particles with a Fermi-like momentum diffusion coefficient  $D_p \propto p^2$ . This is true for both  $\beta \lesssim 1$  and  $\beta \gtrsim 1$  and is thus a robust property of charged particles interacting with low frequency MHD turbulence. We have restricted our analysis to particles whose (relativistic) cyclotron frequencies are larger than the frequencies of the turbulent fluctuations. In practice this limits our analysis to particles that are not too relativistic.

Our key analytic result is that nonlinear broadening of quasi-linear resonances implies that slow modes in strong MHD turbulence can interact efficiently with relativistic particles, despite being unable to satisfy the linear resonance condition (see Chandran 2000 for a similar result). **In particular, resonance broadening allows long wavelength turbulent magnetic field compressions satisfying  $k_{\parallel} c \lesssim \omega_{nl}$  to accelerate particles, where  $\omega_{nl}$  is the non-linear decay rate of the turbulence at a given scale.**

Because slow modes tend to be energetically more important than fast modes in subsonic turbulence, this suggests that interactions with slow modes may dominate the overall particle acceleration by low-frequency, weakly compressible MHD turbulence. This is contrary to the standard quasi-linear theory results in the literature (e.g., Achterberg 1981). However, the particle acceleration efficiency by fast modes depends sensitively on their turbulent power spectrum, which is not fully understood. In particular, if the fast mode spectral index is  $\alpha \sim 3/2$  (which is not the case in our simulations, though it is suggested by some studies), fast modes may be more efficient than slow modes at accelerating particles even if their total energy density is smaller (see eq. 9).

For relativistic particles, momentum diffusion of the form  $D_p \propto p^2$  produces a power-law spectrum  $dN/dp \propto p^{-1}$  so long as the acceleration time of particles (which is independent of particle energy) is shorter than the radiative loss timescale and the escape time from the acceleration region (Blandford & Eichler 1987). The total energy in the accelerated particle population depends on the efficiency with which ‘seed’ relativistic particles are created. Because suprathermal particle acceleration is inefficient for non-relativistic particles interacting with MHD turbulence (Lynn et al. 2012), it is not clear if the net acceleration efficiency (by turbulent

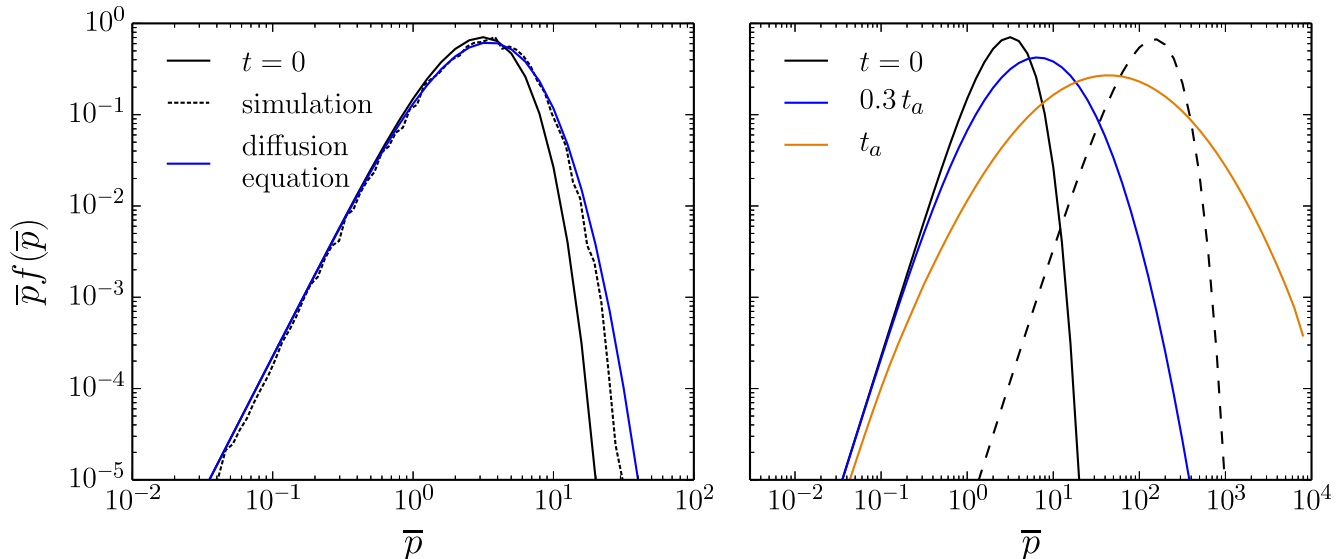


FIG. 4.— Evolution of the particle distribution function due to interaction with MHD turbulence. *Left panel:* Comparison of test particle simulations (dotted) with the numerical solution of the time dependent diffusion equation (solid blue) using a momentum diffusion coefficient given by  $D_{\bar{p}} \equiv \bar{p}^2/t_a$ , where  $t_a$  defines the acceleration time and is derived from the test particle results. The particles are initially thermal and isotropic, and evolve over a long time baseline ( $t = 20 L/c_s$  vs.  $1 L/c_s$  for the calculations used to measure the diffusion coefficient). The numerical solution of the diffusion equation is shown at the same time and is in good agreement with the direct evolution of the test particles. *Right panel:* Longer-time evolution of the numerical solution of the diffusion equation. The particles gain energy exponentially, with an e-folding time of approximately  $0.25 t_a$ . On a comparable timescale, the distribution function develops a significant non-thermal tail. For comparison, we also plot a thermal distribution with the same energy as the final distribution (black dashed curve), which highlights the substantial non-thermal tail at high energies.

mechanisms alone) will be substantial for plasmas with non-relativistic temperatures, because the turbulence itself does not self-consistently seed relativistic particles. By contrast, for relativistically hot plasmas, the formation of a non-thermal tail of relativistic particles by the mechanism studied here is likely to be quite efficient. One particularly important application of our results is thus to accretion flows onto black holes, where the electrons can in some cases have  $kT \gtrsim m_e c^2$  even though the disk turbulence itself is non-relativistic.

### 5.1. Implications for Black Hole Accretion Flows

Weakly compressible MHD turbulence is generic in black hole accretion flows as a consequence of the non-linear evolution and saturation of the magnetorotational instability (Balbus & Hawley 1998). Non-thermal particle acceleration by such turbulence is of particular astrophysical interest in at least two circumstances. First, at low accretion rates onto a black hole or neutron star, the accretion flow can adopt a low-collisionality state in which much of the emission can be dominated by a non-thermal population of electrons, if such a population is present (e.g., Yuan et al. 2003). Secondly, in luminous radiatively efficient accretion flows, non-thermal emission from the disk surface layers (a “corona”) can contribute significantly to the synchrotron and high energy inverse Compton emission. We briefly discuss the implications of our results for these applications.

The momentum diffusion coefficient calculated in §2 and Figure 3 corresponds to a rate of energy gain given by

$$\dot{E}_{\text{acc}} \sim \frac{c D_p}{p} \equiv A p \frac{v_A^2}{L} \left( \frac{\delta B_{\parallel}}{B_0} \right)^2 \quad (12)$$

where  $A$  is a dimensionless coefficient that encapsulates the efficiency of the particle acceleration and can be calibrated using our test particle simulations. In particular, Figure 3 corresponds to  $A \sim 1/3$  for  $c/v_A \sim 10 - 100$ , the values expected in the inner regions of accretion disks around black holes. The exact value of  $\delta B_{\parallel}/B_0$  in accretion disk turbulence is somewhat uncertain. For the  $\beta \sim 10 - 100$  conditions expected,  $\delta B_{\parallel} \sim 0.3 B_0$  is plausible. However, the exact value depends in part on the effect of collisionless damping on the compressibility of accretion disk turbulence, which is not well understood. Moreover, small-scale fluctuations generated by the mirror instability may contribute significantly to the magnetic field compressions in collisionless disks (Kunz et al. 2014; Riquelme et al. 2014).

The acceleration of particles by disk turbulence requires that the acceleration time is shorter than the viscous time. Given the acceleration rate in equation 12 this is likely achieved in the inner regions close to the black hole. In addition, the acceleration of particles by disk turbulence is limited by radiative losses, in particular synchrotron and inverse Compton emission. Focusing on the former, we find that the maximum Lorentz factor of accelerated electrons is given by

$$\gamma_{\text{max}} \sim A \left( \frac{m_e}{m_p} \right) \tau_T^{-1} \left( \frac{\delta B_{\parallel}}{B_0} \right)^2 \quad (13)$$

where  $\tau_T \equiv \sigma_T n_e L$  is the Thompson optical depth across the outer scale of the turbulent fluctuations  $L$ . Equation 13 implies that non-thermal emission from accelerated electrons is likely to be particularly important in low-luminosity systems where  $\tau_T \ll 1$ . As a concrete example, in models of the emission from Sgr A\*,



$\tau_T \sim 10^{-5} - 10^{-6}$  (e.g., Yuan et al. 2003; Mościbrodzka et al. 2009) so that  $\gamma_{\max} \sim 100$ . This implies that the particle acceleration found here may substantially modify the electron distribution function for electrons that emit in the mm-infrared. This is particularly important to understand in the context of interpreting the variable infrared emission and resolved mm images of Sgr A\* (e.g., Doeleman et al. 2008; Do et al. 2009). In the near future, more detailed calculations of test particle electron acceleration in shearing box simulations can be used to quantify the uncertain dimensionless coefficient  $A$  in the above acceleration efficiency.

A second potential application of our results is to high energy emission from luminous accreting black holes, which can be produced by a combination of thermal and non-thermal processes. However, phenomenologi-

cal models of this emission suggest that  $\tau_T \sim 0.1 - 1$  in the emission region (Haardt & Maraschi 1991; Esin et al. 1997). As a result, it is unlikely that the particle acceleration found here is sufficiently rapid to compete with radiative losses by synchrotron and inverse Compton emission.

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